

Exercise 2.4, also Study Note MFE-01-17, Problem No. 1, and Dr. Ostaszewski's online exam FM exercise No. 119 posted August 25, 2007

Which statement about zero-cost purchased collars is FALSE?

- A. A zero-width, zero-cost collar can be created by setting both the put and call strike prices at the forward price.
- B. There are an infinite number of zero-cost collars.
- C. The put option can be at-the-money.
- D. The call option can be at-the-money.
- E. The strike price on the put option must be at or below the forward price.

Solution.

- A is true. A purchased zero-width collar equals a portfolio of short call and long put, with these two options having the same strike price. This is equivalent to a short forward position, with the price at which the forward is entered into being the common strike price of both options. But a forward transaction is costless if its price is the forward price.
- B is also true, because once we create a zero-cost collar, by varying strike prices of both the call and the put we can create infinitely many positions with zero cost (note that B does not assume that the collar is zero-width, in fact there is no assumption about the strike prices at all).
- C is true, the put option can be at-the-money, and then the strike price of the call option needs to be chosen appropriately. However, in a zero-cost collar there are some restrictions on the range of strike prices for the call and the put, as we will see when we discuss D.
- D is false. Recall that a zero-cost zero-width collar is created at the exercise price of both the call and the put. If we keep the collar at zero-width, but lower the price from forward to spot (assuming that interest rate is positive, $S < F = Se^{rt}$), call premium will go up, while put premium will go down. To make the collar zero-cost we would have to
 - Either lower the call premium by increasing its exercise price, while keeping the put at the money (this is what happened in C),
 - Or increase the put premium by increasing its exercise price, while keeping the call at the money. But in a collar, put exercise price is supposed to be below the call exercise price.
- E is true. If the put strike price X_p is above the forward price $F = AV(S)$, where AV denotes the accumulated value, and X_c is the call strike price, then we would have $X_c \geq X_p > F$. Let C' be the price of the call with exercise price X_p . Then

$$0 = C - P \leq C' - P = S - PV(X_p) = PV(F) - PV(X_p) = PV(F - X_p) < 0.$$

This is a contradiction. Therefore, for a zero-cost collar, the put strike price cannot exceed the forward price. E is true.

Answer D.